Spatio-temporal complexity of power-grid frequency recordings in the Nordic grid

Leonardo Rydin Gorjão

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Dept. of Computer Sciences, Metropolitan University of Olso, Norway | THP, University of Cologne, Germany

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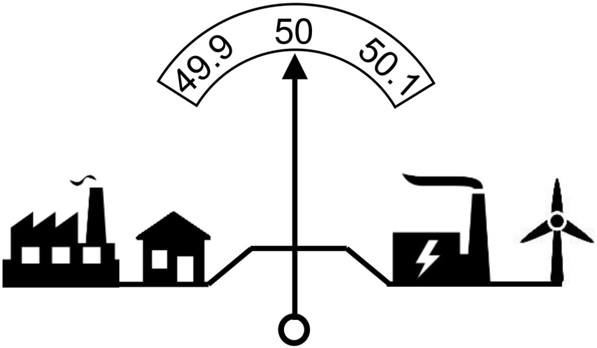
Power-grid frequency recordings

Excerpts from February 2019



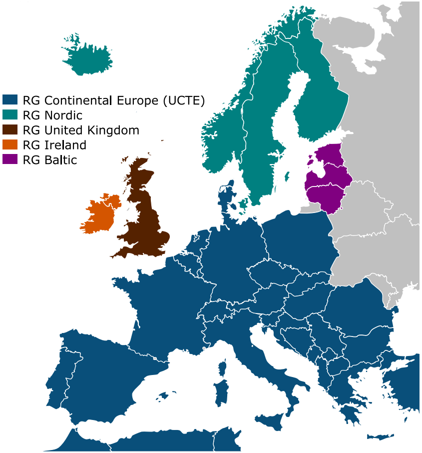
Short introduction to power grids

A focus on the Nordic Grid

In Europe there are five main power grids

- Continental Europe
- Nordic Grid
- National Grid (Great Britain)
- Baltic Grid
- EirGrid (Ireland)

LRG, B. Schäfer, D. Witthaut, and C. Beck, *Spatio-temporal complexity of power-grid frequency fluctuations*, New Journal of Physics **23** 073016 (2021)
`doi.org/10.1088/1367-2630/ac08b3`



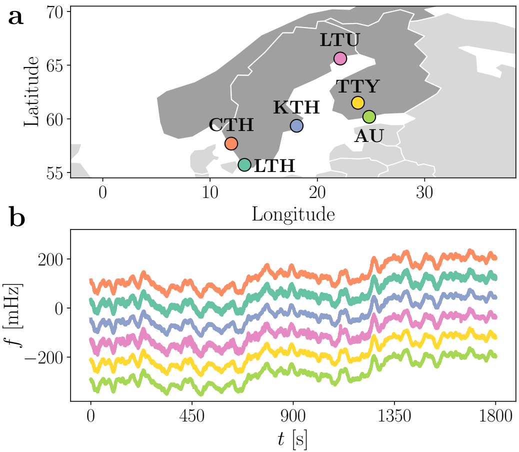
Short introduction to power-grid frequency
Recordings in the Nordic Grid

Nordic Grid: 6 synchronous recordings

- CTH Chalmers University of Technology Gothenburg
- LTH Faculty of Engineering, Lund University
- KTH Royal Institute of Technology Stockholm
- LTU Luleå University of Technology
- TTY Tampere University of Technology
- AU Aalto University

Recorded with 0.02 sec. resolution

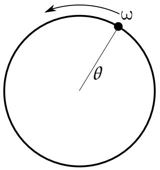
data: `https://power-grid-frequency.org/`



† M. S. Almas, M. Baudette, L. Vanfretti, S. Løvlund and J. O. Gjerde *2014 IEEE PES General Meeting, pp 1–5*

A single generator/motor is naïvely described via

$$\frac{\mathrm{d}\theta_i}{\mathrm{d}t} = \omega_i,$$
$$M_i \frac{\mathrm{d}\omega_i}{\mathrm{d}t} = -D_i\omega_i + P_i^m - P_i^{el},$$
$$T_i \frac{\mathrm{d}E_i}{\mathrm{d}t} = E_i^f - E_i + (X_i - X_i')I_i,$$

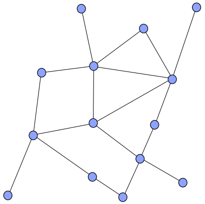


$$P_i^{el} = E_i \sum_{j=1}^{N} E_j B_{i,j} \sin(\theta_i - \theta_j),$$
$$I_i = \sum_{j=1}^{N} E_j B_{i,j} \cos(\theta_i - \theta_j),$$

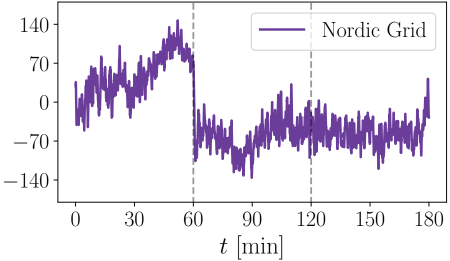
with M_i the inertial mass, D_i the damping constant, P_i^m the mechanical power (generated or consumed), P_i^{el} the exchanged electrical power, E_i^f the field flux, $(X_i - X_i')$ the different of static and transient reactance, and I_i the current through the machine/motor.

When modelling a power grid, we have *models* and *recordings/data.*

A "networked" power-grid model



A power-grid frequency recording



How to best describe power-grid frequency recordings?

Short introduction to power-grid frequency

A stochastic approach

A naïve model for the power-grid frequency f_i (angular velocity ω_i) is given by

$$M_i \mathrm{d}\omega_i = -D_i \omega_i \, \mathrm{d}t + \Delta P_i \mathrm{d}t + \epsilon_i \, \mathrm{d}W_i(t),$$

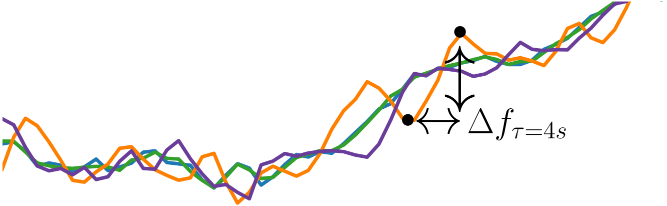
where D_i is the *damping* of machine i, M_i is the inertial mass of machine i, ΔP_i is the power mismatch, and W_i is a Wiener process with amplitude ϵ_i.

Three notes:

- the ratio D_i/M_i need *not* be constant
- the noise amplitude ϵ_i/M_i need *not* be constant
- $\mathrm{d}W_i(t)$ can be a more complicated process (non-stationary, multifractal, etc.)

We focus on the increment statistics of power-grid frequency recordings

Increment statistics: $\Delta f_{i,\tau}(t) = f_i(t+\tau) - f_i(t)$



We focus on the change in the noise characteristics

$$\mathrm{d}\omega_i = -D_i/M_i\omega_i\,\mathrm{d}t + \Delta P_i/M_i\mathrm{d}t + \boxed{\epsilon_i/M_i\,\mathrm{d}W_i(t)}$$

Quantifying fluctuations
Recordings in the Nordic Grid, 2013

Nordic Grid: 6 synchronous recordings

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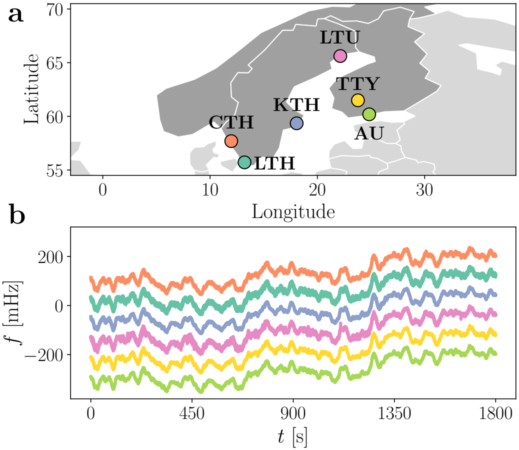

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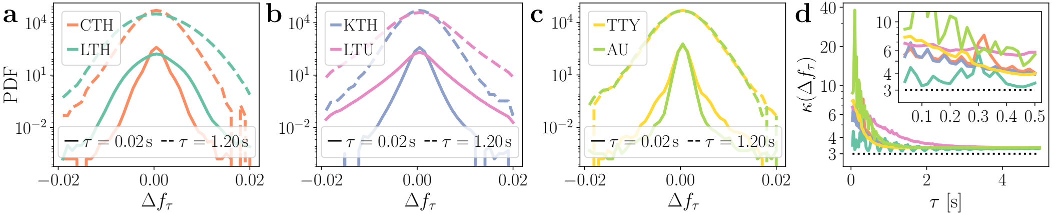
We focus on the increments of power-grid frequency. The probability distributions of the increments show



The kurtosis (heavy-tailedness) is given by

$$\kappa(X) = \mathbb{E}\left[\left(\frac{X-\mu_X}{\sigma_X}\right)^4\right] = \frac{\mathbb{E}\left[(X-\mu_X)^4\right]}{\left(\mathbb{E}\left[(X-\mu_X)^2\right]\right)^2}, \quad (1)$$

We note that all distributions are leptokurtic ($\kappa > 3$).

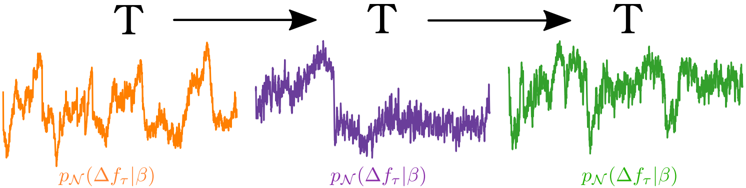
Superstatistics in the Nordic Grid

Pictorial representation

We take a simple assumption that the statistics of the distribution is given by a superposition of statistics (super-statistics), given by

$$p(\Delta f_\tau, \tau) = \int_0^\infty f(\beta) \boxed{p_\mathcal{N}(\Delta f_\tau, \beta)} \mathrm{d}\beta$$

Pictorially, this means that



T is the *long superstatistical time*.

Superstatistics in the Nordic Grid

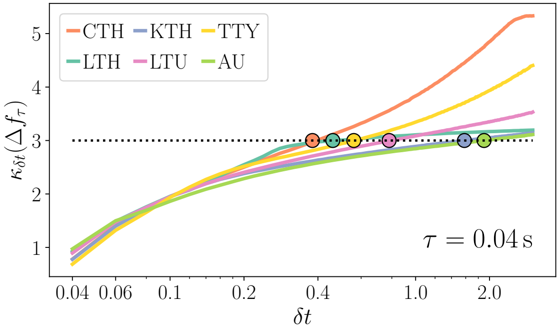
Finding local equilibrium

We consider the underlying equilibrium statistics is Gaussian distribution

$$p_{\mathcal{N}}(\Delta f_\tau|\beta) = \sqrt{\frac{\beta}{2\pi}} e^{-\frac{1}{2}\beta\Delta f_\tau^2}.$$

We estimated the *long superstatistical time* T by checking when a local equilibrium is attained, i.e.

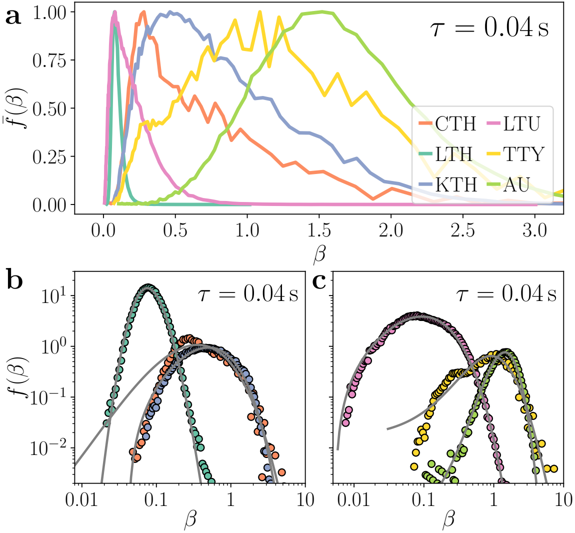
$$\kappa_{\delta t}(\Delta f_\tau) \equiv 3.$$

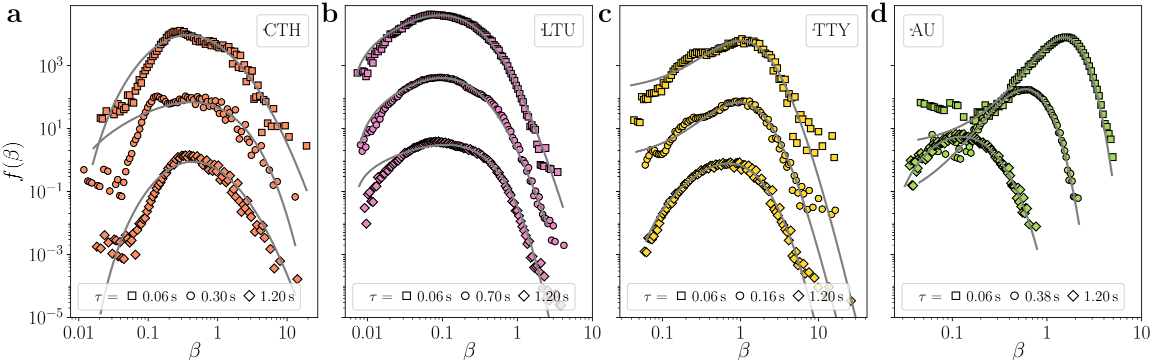


We obtain the distributions $f(\beta)$ of the volatilities β, which we **cannot** precise.

Location	T	$f_{\log\mathcal{N}}(\beta)$	$f_{\Gamma}(\beta)$	$f_{\mathrm{inv}\Gamma}(\beta)$	$f_{F}(\beta)$
CTH	0.38 s	0.0344 5.5%	**0.0326** **0.0%**	0.0473 45.0%	0.0384 17.7%
LTH	0.46 s	0.0178 124.4%	0.0988 1141.9%	0.0081 1.3%	**0.0080** 0.0%
KTH	1.58 s	0.0287 59.4%	**0.0180** **0.0%**	0.0319 77.2%	0.0184 2.1%
LTU	0.78 s	0.0251 106.8%	**0.0121** **0.0%**	0.0299 146.7%	0.0147 1.2%
TTY	0.56 s	0.0249 4.2%	0.0342 43.1%	**0.0239** **0.0%**	0.0240 0.4%
AU	1.90 s	0.0023 43.5%	0.0048 198.0%	**0.0016** **0.0%**	0.0024 46.7%

(values indicate the Kullback–Leibler divergence)



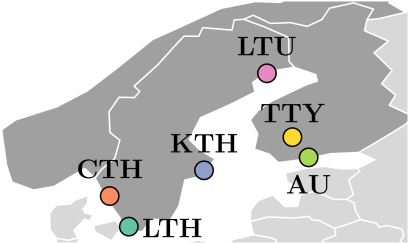


Increment statistics: $\Delta f_{i,\tau}(t) = f_i(t+\tau) - f_i(t)$

Superstatistics in the Nordic Grid
Entropic indices

To quantify the fluctuation of fluctuation of power-grid frequency, utilise the *entropic index* q

$$q = \frac{\langle \beta^2 \rangle}{\langle \beta \rangle^2}, \quad \text{with } q = 1 \Longleftrightarrow p(\Delta f_\tau) \equiv p_{\mathcal{N}}(\Delta f_\tau, \beta) \quad (f(\beta) = \delta(\beta))$$



Which results in the *entropic indices* q:

Location	$\tau = 0.02\,\mathrm{s}$	$\tau = 0.04\,\mathrm{s}$	$\tau = 1.20\,\mathrm{s}$
CTH	1.772	1.838	1.799
LTU	1.572	1.618	1.568
TTY	1.506	1.492	1.397
AU	1.119	1.103	1.253

We also find that ~ 2 secs. superstatistics loses validity.

Outlook

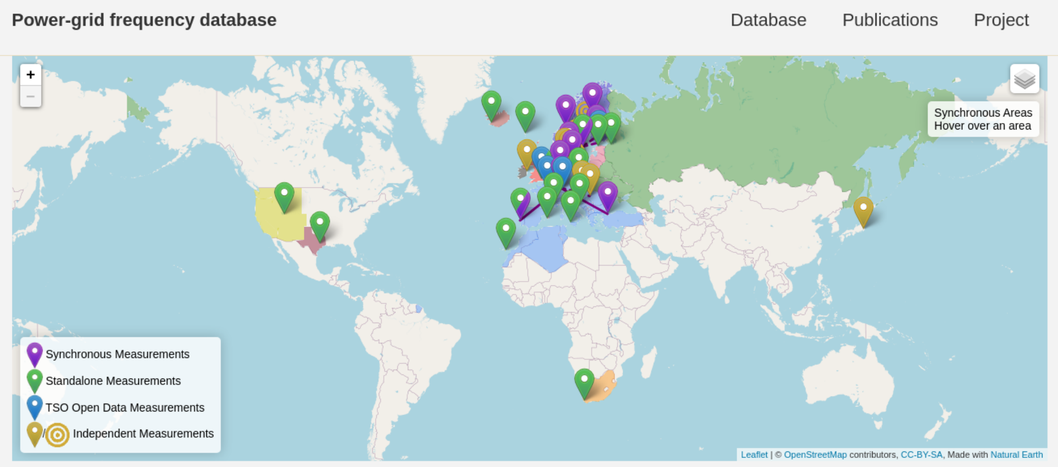
We have observed that, at short time scales (<2 secs):

- Power-grid frequency increments are leptokurtic ($\kappa > 3$).
- All recordings have a different long time scale.
- Is is not straightforward to identify a governing superstatistics.
- All q indices vary, yet remain constant in each location for different incremental lag τ.
- Superstatistics loses validity at ~ 2 secs (not shown).

We gather that, at short time scales (<2 secs):

- Power-grid frequency recordings reflect *mainly* local properties of the grid
- All recordings show signs of multifractality.
- Yet, we have **not** identified the cause for these differences.

Power-grid frequency database



data: `https://power-grid-frequency.org/`

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Thank you for your attention

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