Motivation

Renewable-energy sources are scattered and fluctuating. Their increasing penetration places the issue of electrical grid stability in the wider problem of stability of noisy coupled dynamical systems.

\{electrical grid stability\} \subset \{perturbed dynamical systems\}.

We assess the time needed for a dynamical system to be destabilized, based on the noise’s parameters.

In our approach, a larger amount of systems, issue of electrical grid stability

The networks considered are:

- \(C_1\) - The cycle of length \(n = 83\) vertices;
- \(C_3\) - The cycle with first- and third-neighbors with \(n = 83\) vertices;
- \(UK\) - The UK transmission network composed of \(n = 120\) vertices and 365 edges;
- \(SW\) - A small world network with \(n = 280\) vertices.

The criterion Eq. (3) gives a good parametric estimate of the boundary between the regions \(U\) and \(S\).

Remarking the following asymmetry does not depend on inertia,

\[
\lim_{t \to \infty} \langle \theta(t) \theta(t) \rangle = \frac{\delta P}{m + d} \quad \text{for}\ \theta_0 = \frac{\pi}{4}.
\]

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The model

We consider the second-order system of \(n\) coupled oscillators, which represent the swing equations in the lossless line approximation:

\[
m \ddot{\theta}_i + d \dot{\theta}_i = P(t) - \sum_{j=1}^{n} b_{ij} \sin(\theta_i - \theta_j),
\]

\(b_{ij}\), \(\theta_i\) and \(\dot{\theta}_i\) are respectively the inertia and damping of each oscillator; \(P(t)\) are the time-varying natural frequencies or power injections/consumptions; \(b_{ij}\) are the elements of the weighted adjacency matrix of the interconnection graph.

Decomposing, \(P = P^{(0)} + \delta P(t)\) and \(\theta = \theta^{(0)} + \delta \theta(t)\) and linearizing Eq. (1), we get

\[
m \ddot{\delta \theta} + d \dot{\delta \theta} = \delta P - L \langle \delta \theta^2 \rangle \delta \theta,
\]

where \(L_j = \sum_{i \neq j} b_{ij} \cos(\theta_i - \theta_j), \ i \neq j\) is a weighted Laplacian matrix.

We apply an additive random colored noise to all natural frequencies, giving an estimate of the parameter domain where the system is unlikely to be destabilized. The time needed to see such large excursions is estimated as

\[
\tau_{\text{esc}} = \left( \int_{\Delta \nu} P(\delta \theta) d\delta \theta \right)^{-1},
\]

which is superexponential (red crosses).

Inertia

Comparing the cases \(m > 0\) and \(m = 0\), our analytical prediction and the simulations both conclude that inertia almost always destabilizes the system.

In the context of electrical network, however, the value of \(\theta_0\) is very large compared to the time scales of the network. Such system then evolves in a parameter region where the difference is negligible.

Conclusion

We proposed a method to assess the time needed for a system to leave its basin of attraction. This criterion is efficient to compute as it mainly relies on the inversion of a Laplacian matrix. Under our assumptions, for a sufficiently long time, any system ends up escaping its basin. But the time needed for this increases superexponentially, exceeding any realistic time for any practical application.

References


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